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Contribution to flow characterisation through porous media

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Abstract

This paper reports a theoretical and experimental research in the field of mass transport through porous media. Thermodynamics was employed to model the change of mass inside porous media and the fluid flow through porous media. An experimental study was carried out to provide a thermodynamic chart of a porous medium and to infer the parameters required by the theoretical approach. The effect of the moisture content and temperature on fluid transport properties was also investigated. © 2000 Elsevier Science Ltd. All rights reserved.

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1. Introduction

The study of transport phenomena in porous media has attracted the attention of theorists and experimentalists in recent years, due to its scope in various fields of engineering and environmental sciences [1]. Despite the existence of a lot of important works on this topic, the case of an adsorbed fluid within a porous medium and its influence on the mass transport properties of the medium has been a long-standing problem [2–4]. It is clear that a correct understanding of these topics, both from the theoretical and the experimental point of view, is essential for a better characterisation of the mass transfer in porous media [3–5].

This paper aims to further understanding of mass transfer in porous media. Thermodynamics is employed to model the mass change inside a porous media as a function of intensive quantities. This approach, together with the fluid motion equation, provides for a simple derivation of the fluid transport properties of porous media, and for the analysis of fluid flow dependence on the fluid adsorption within the porous media.

An experimental study was carried out in order to obtain a thermodynamic chart of a porous medium and the parameters required by the theoretical approach. In this study, the effect of moisture content and temperature of a porous medium on the fluid transport properties has also been investigated.

2. Theory

A porous medium is composed of a solid phase (matrix) and a fluid. To study the interaction between the matrix and the fluid, we can formulate the problem in the framework of thermodynamics.

The principle of conservation states that the change in the internal energy of a system is a consequence of the thermal, mechanical and diffusive interactions between the subsystem and its surroundings. This

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Nomenclature

A	area (m^2)
F	force (N)
M	mass (kg)
P, p	pressure (Pa)
S	entropy (J K^{-1})
T	temperature (K)
r	position (m)
U	internal energy (J)
u	velocity (m s^{-1})
V	volume (m^3)

Greek symbols

ε	porosity
Λ	quantity like volume, entropy, or mass
μ	dynamic viscosity (Pa s)
θ	volumetric moisture content
ρ	density (kg m^{-3})
κ	fluid conductivity (m^{3}s kg^{-1})
ς_{S}	parameter defined in Eq. (13) (kg K^{-1})
ς_{V}	parameter defined in Eq. (14) (m s^{-2})
ω	chemical potential (J kg^{-1})

Subscripts

ef	effective
m–f	matrix–inside fluid

statement forms the first law of thermodynamics which, for a system composed of n subsystems, can be mathematically expressed as

$$\delta U = \sum_{i=1}^{n}(\delta U_i) = \sum_{i=1}^{n}(\mathbf{F}_i\delta\mathbf{r}_i + \zeta_i\delta\mathbf{A}_i - P_i\delta V_i + T_i\delta S_i + \omega_i\delta M_i) \quad (1)$$

where U is the internal energy, T the temperature, P the pressure, V the volume, M the mass, ω the chemical potential, S the entropy, and ζ, $\mathbf{F}$ and $\mathbf{r}$ are the vectors for superficial stress, force, and position, respectively.

Each i-subsystem can exchange quantities like energy, entropy, volume and mass with its surroundings. The budget equation for a quantity Λ is then

$$\sum_{i=1}^{n}(\delta\Lambda_i) = \delta\Lambda_{\mathrm{t}} \quad (2)$$

where $\delta\Lambda$ represents the variation of the quantity Λ in the i-subsystem and $\delta\Lambda_{\mathrm{t}}$ the creation or destruction of the quantity Λ in the transfer process.

In the case of conservative quantities like energy, volume and mass we only have to write

$$\sum_{i=1}^{n}(\delta\Lambda_i) = 0 \quad (3a)$$

but for entropy, the second law of thermodynamics imposes that

$$\sum_{i=1}^{n}(\delta S_i) = \delta S_{\mathrm{t}} \quad (3b)$$

where S_i is the entropy in i-subsystem.

The exchange of Λ between a specific subsystem (subsystem α) and the remaining thermodynamic subsystems can be written as

$$\delta\Lambda_{\mathrm{t}} = \sum_{i=1}^{n}(\delta\Lambda_i - \delta\Lambda_\alpha) \quad (4)$$

Taking into account Eqs. (1)–(4) we can write

$$T_\alpha\delta S_{\mathrm{t}}\sum_{i=1}^{n}(\mathbf{F}_i - \mathbf{F}_\alpha)\delta\mathbf{r}_i + \sum_{i=1}^{n}(\zeta_i - \zeta_\alpha)\delta\mathbf{A}_i - \sum_{i=1}^{n}(P_i - P_\alpha)\delta V_i + \sum_{i=1}^{n}(T_i - T_\alpha)\delta S_i + \sum_{i=1}^{n}(\omega_i - \omega_\alpha)\delta M_i \geq 0 \quad (5)$$

where $T_\alpha\delta S_{\mathrm{t}}$ is the so-called availability function.

Eq. (5) describes the thermal, mechanical and diffusive interactions between the subsystem α and the remaining i-subsystems.

A porous medium (a subsystem composed of a matrix and its inside fluid) submerged in an external fluid (subsystem external fluid) composes the thermodynamic system, Π, under study. Assuming that the superficial stress of the fluid within or outside the matrix is equal, Eq. (5) becomes

$$T(\delta S_{\mathrm{t}}) = \delta\mathbf{F}\cdot\delta\mathbf{r}_{\mathrm{m-f}} - \delta P\,\delta V_{\mathrm{m-f}} + \delta\omega\,\delta M_{\mathrm{m-f}} + \delta T\,\delta S_{\mathrm{m-f}} \quad (6)$$

where $\mathbf{F}$ represents the vector force resulting from mechanical effects, gravity, etc. The subscript m–f means matrix–inside fluid.

The mass change in the subsystem matrix–inside fluid is given by

$$\delta M_{\text{m–f}} - \varsigma_\tau T = \varsigma_V \delta P - \varsigma_V \delta p + \varsigma_S \delta T \tag{7}$$

with

$$\begin{aligned} \delta p &= (\mathbf{I}\cdot\delta\mathbf{F})/\delta A_{\text{m–f}} \\ \varsigma_V &= (\delta V_{\text{m–f}}/\delta\omega) \\ \varsigma_S &= -(\delta S_{\text{m–f}}/\delta\omega) \\ \varsigma_\tau &= (\delta S_t/\delta\omega) \end{aligned}$$

where the coefficient ς_V defines the rate between the change in volume of the subsystem matrix–inside fluid and the change in chemical potential of the whole system Π; the coefficient ς_S defines the rate between the entropy change of the subsystem matrix–inside fluid and the change in chemical potential of the whole system Π; $\mathbf{I}$ represents the unit vector.

Eq. (7) relates the mass change in the subsystem matrix–inside fluid with intensive quantities (p, P, T), generalising the relation obtained from Ref. [3].

Equilibrium is a state in which quantities are not time-dependent and in which there are no temperature, pressure and velocity gradients, thus, $\delta S_t = 0$. Near equilibrium, it is reasonable to consider that the last term on right-hand side of Eq. (7) is small enough to be neglected. Therefore, Eq. (7) yields

$$(\delta V_{\text{m–f}})\delta\rho = \varsigma_V\, \delta P - \varsigma_V\, \delta p + \varsigma_S \delta T \tag{8}$$

where

$$\delta\rho = (\delta M_{\text{m–f}}/\delta V_{\text{m–f}})$$

2.1. Fluid flow and driving potentials

Considering the mass exchange between the subsystem matrix–inside fluid and the subsystem external fluid in spatial coordinates $(\delta x, \delta y, \delta z)$, the thermodynamic equilibrium pressure equation yields

$$\nabla P = \nabla p + (V_{\text{m–f}}/\varsigma_V)\nabla\rho - (\varsigma_S/\varsigma_V)\nabla T \tag{9}$$

Insertion of Eq. (9) into the equation of motion gives a new insight to the fluid flow phenomenon. The basic equation of motion for porous media is the Darcy law [1]. To be consistent with a no-slip condition, Brinkman [6] modified the Darcy law by adding the Laplacian of the velocity, resulting in

$$\mathbf{u} - \kappa\mu_{\text{ef}}\nabla^2\mathbf{u} = -\kappa\nabla P \tag{10}$$

where κ represents the fluid conductivity and μ_{ef} the effective viscosity (the ratio between the dynamic viscosity and the porosity).

Substitution of Eq. (9) into (10) yields

$$\mathbf{u} - \kappa\mu_{\text{ef}}\nabla^2\mathbf{u} = -\kappa\nabla p - \kappa_\rho\nabla\rho + \kappa_T\nabla T \tag{11}$$

with

$$\kappa_\rho = \kappa\left(\frac{V_{\text{m–f}}}{\varsigma_V}\right) \tag{12a}$$

$$\kappa_T = \kappa\left(\frac{\varsigma_S}{\varsigma_V}\right) \tag{12b}$$

where κ_ρ represents the fluid conductivity for fluid transport due to density gradients and κ_T the fluid conductivity for fluid transport due to temperature gradients.

Eq. (11) shows how the fluid velocity is related to the gradients of p, ρ and T, through the fluid conductivity κ and the parameters ς_S and ς_V.

According to Eq. (11), in the absence of density and temperature gradients $(\nabla p \neq 0)$, the fluid flow is independent of ς_S and ς_V, thus only depending on κ. If ∇p and ς_S are null, the density gradient is the only cause for the fluid flow. If ∇p is null and $V_{\text{m–f}} \ll \varsigma_V$, the fluid flow is only due to the temperature gradient.

2.2. Parameters ς_S and ς_V

Parameters ς_S and ς_V are described by Eq. (8) and can be related to ρ, P, p and T.

Under isobaric conditions, Eq. (8) becomes

$$\varsigma_S = \delta M_{\text{m–f}}/\delta T \tag{13}$$

and in the absence of wind induced pressure, for isothermal conditions, Eq. (8) becomes

$$\varsigma_V = \delta M_{\text{m–f}}/\delta p_v \tag{14}$$

where p_v is the vapour pressure of the surrounding air.

3. Experimental study

Any model describing a physical phenomenon is meaningless unless we can determine the values of the coefficients appearing in it. In this study, the fluid conductivity κ, parameters ς_S and ς_V, as well as a thermodynamic chart of a porous medium were determined.

3.1. Moisture intrusion and parameters ς_S and ς_V

In a porous medium, the moisture intrusion (or hygroscopic moisture content) is related to the external conditions of humidity and temperature [7]. Parameters ς_S and ς_V on porous media are in turn related to the moisture intrusion and can be determined by Eqs. (13) and (14), using adsorption/desorption isobars and adsorption/desorption isotherms. That is, these quantities can be determined from the mass change of the medium sample, using as initial conditions a state in which the medium is only composed by a solid matrix free from fluid.

3.1.1. Measurements

The above-mentioned quantities were determined by laboratory experiments using sand (particle size 180–460 μm, porosity 0.29 and density 1450 kg m^{-3}, in the dried state) as the medium sample.

The mass change measurement of a sand sample initially free from fluid was measured using a precise analytical scale with an accuracy of 0.01×10^{-6} kg.

To have samples free from inside fluid (initial conditions), several sand columns of 8 cm^3 each were placed in an oven at a temperature of 104°C, in which, vacuum was continuously applied [8]. The sample's fluid loss was determined by continuously monitoring the decrease in their weight. Before 10 days, the sample's matrix was completely free from fluid.

Each sand sample (matrix free from fluid) was then placed in a climatised chamber where it was subjected to various fixed temperature and humidity combinations. The totality of the experiments covered temperatures and humidities ranging between 5°C and 40°C, and 5% and 95%, respectively. In all cases, the equilibrium between the sample and the chamber atmosphere was reached within 3 weeks.

3.2. Fluid conductivity κ

In the apparatus depicted in Fig. 1, a fan (1.5 kW, 2800 rpm) forces air through the test column containing sand, in order to create pressure drops. In a stationary state, the fluid conductivity of the sand sample can then be determined using the Darcy law [9]. The airflow supplied by the fan (B) is regulated by controlling the rotational speed of the fan. The volumetric flow rate is measured by the frequency of rotation of a calibrated measuring fan (A). The flow is conducted to the air inlet (C) in order to create a uniform velocity profile. To assist this effect, the air is forced through a pressure equaliser (D), a perforated metal structure (1.5 mm diameter holes; porosity 0.23), before reaching the test column. The test column (E) consists of a 0.40 m long glass cylinder with an internal diameter 0.10 m. Pressure taps are located at levels (F) and (G). The pressure drop is measured using two membrane pressure transducers (accuracy 0.01 Pa), having one part connected to the pressure tap (F) and the other to the pressure trap (G).

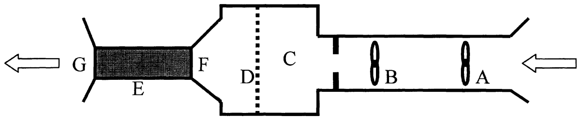

Fig. 1. Schematic representation of the apparatus used to measure the fluid conductivity κ of a sand sample.

3.2.1. Measurements

The test apparatus was placed in a climatised chamber, as described in Section 3.1.1, in order to study how the sand conductivity κ is related to the sample's temperature and moisture content.

To make the sand sample reach the desired temperature and moisture content, the following procedures were executed. First, a sample free from inside fluid was obtained by closely following the procedure described in Section 3.1.1. Afterwards, the sample was placed in a climatised chamber until an equilibrium between the sand and the chamber atmosphere was reached (8–9 weeks). Fixed temperature and humidity combinations ranged within the same values as in Section 3.1.1. Finally, the pressure drop determinations were performed.

4. Results and discussion

The experimental results are presented in Figs. 2–5. In Fig. 2, a graphical representation of the thermodynamic properties of moist sand (solid matrix and adsorbate) is presented. The data depicted in the chart represent the hygroscopic moisture content in the sand.

Figs. 3 and 4 present parameters ς_V/V and ς_S/V for different temperatures (5–40°C) and moisture contents (2–18), respectively. In Fig. 3, we can see that ς_V/V is dependent on both, the moisture content as well as the temperature. The results shown in this figure lead to the conclusion that ς_V/V is primarily dependent on the

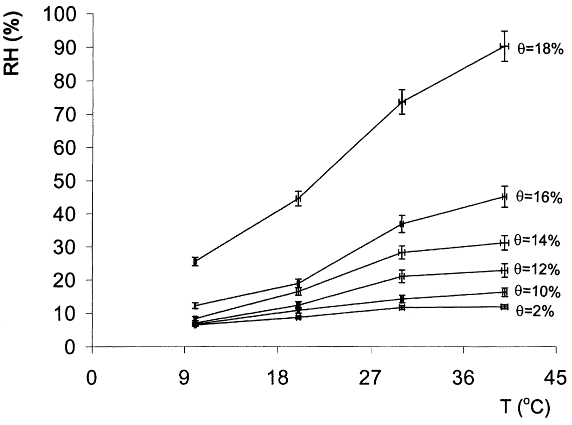

Fig. 2. Thermodynamic chart of moist sand (solid matrix and adsorbate).

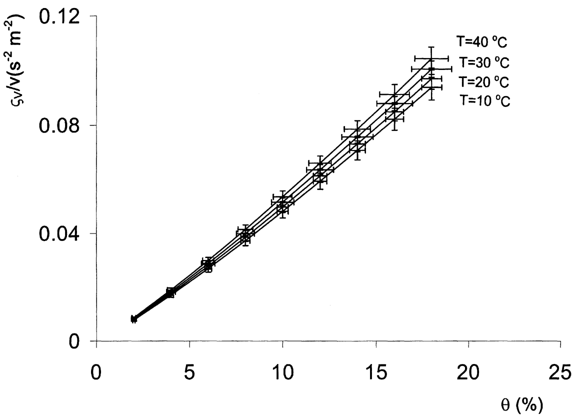

Fig. 3. Dependence of parameter ς_V/V on moisture content for different temperatures.

moisture content, having only a weak variation with temperature.

In Fig. 4 also, we can observe the dependence of ς_S/V on the temperature and moisture content. The small differences of ς_S/V for different moisture content curves of 10–14% ($T < 20$°C) are consistent with the fact that these curves correspond to quasi-saturated sand samples, that is, samples with pores almost completely filled with liquid water.

The influence of moisture content and temperature on the fluid conductivity κ is presented in Fig. 5. This figure shows that the fluid conductivity is very sensitive to both, temperature and moisture content. Neglecting this dependence gives rise to large errors in the κ value determination. As an example, the fluid conductivity value (κ) for a moisture content of 10% is between 53 and 57% lower than that for a moisture content of 18%; the κ value for a temperature of 10°C is between 35 and 46% lower than that for a temperature of 30°C.

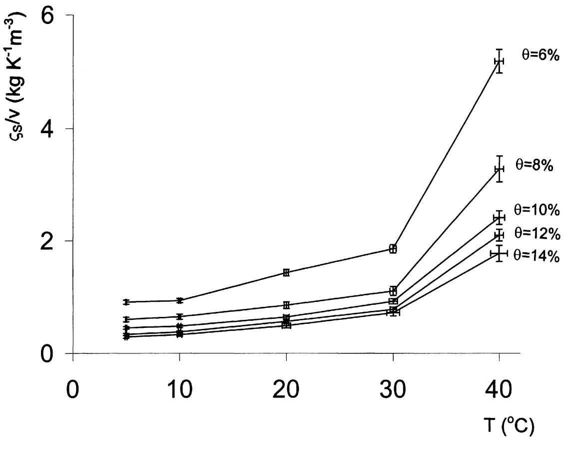

Fig. 4. Dependence of parameter ς_S/V on temperature for different moisture contents.

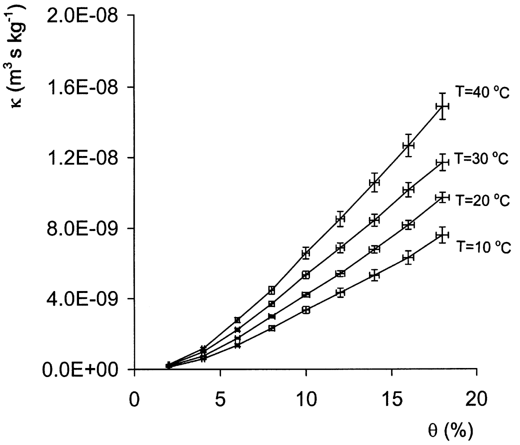

Fig. 5. Effect of moisture content on sand fluid conductivity κ for different temperatures.

The results presented in Figs. 3–5 are consistent with the fact that most of the moisture content of the medium is adsorbed in the inner walls of the pores, and that, the filling of the narrowest pores with liquid water is strongly dependent on temperature and moisture content (Kelvin's law).

5. Conclusion

In this paper, thermodynamics was used for modelling a porous system. From the resulting approach, the mass change within the porous medium (as a function of intensive quantities) was modelled. The insertion of the thermodynamic approach on the equation of motion was used to clarify the fluid transport in porous media. The fluid transport properties were precisely stated and the effect of fluid adsorption on fluid transport was also included. Laboratory experiments were carried out using sand samples as models, in order to obtain the model parameters ς_V and ς_S, fluid conductivity κ, as well as the thermodynamic chart of the sand sample.

Parameter ς_V was found to be mainly dependent on the moisture content. For the sand sample used in this study, the ς_V value was estimated to range between 0.0074 and 0.105 (m s^{-2}). Parameter ς_S and fluid conductivity κ were found to be functions of both temperature and moisture content. For the sand sample used in this study, ς_S and κ values ranged between 0.29 and 5.2 (kg K^{-1}), and 1.18×10^{-10} and 1.49×10^{-8} (m^3 s kg^{-1}), respectively. We must specially emphasise the fluid conductivity dependence on both the temperature and moisture content, since this means that the fluid transmissivity of a porous medium can differ greatly, with different environmental conditions of humidity and temperature. It is important to note that porous media applications in building conditioning can be greatly affected by the severe humidity and temperature dependence of κ.

The experimental results confirm that, in order to obtain a correct value for the fluid conductivities κ_p and κ_T, and to prevent large errors in the determination of fluid transport through porous media, parameters ς_S, ς_V and fluid conductivity κ should be considered as a function of both temperature and humidity.

The information contained in the thermodynamic chart allow us to obtain the moisture content of the porous media for specific conditions of temperature and humidity. This information can be used to choose the best path among different drying/moistening processes, given some defined constraints (for example, less energy consumption).

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